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The Probability Density Function for Two Phase Flow Motions in Fluids

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Abstract

Two mathematical presentations, the Eulerian-Eulerian (EE) and the Lagrangian-Eulerian (LE) presentations are found in the framework of the potential density function (p.d.f.) for the order of flow of multiple phases. A consistent relationship between basic statistical values in EE and LE presentations is found. The core values in these statistical representations show some correlation under the conditions of spatial similarity. Transport traffic congestion statistics for each statistical template are available. The dominant figures for medium weight, medium pressure and second minute speed in relation to these two methods are taken from these transport statistics. In particular, in the representation of EE, p.d.f. formal performance is shown as a natural lead to the integration of the intermediate working figures of the two flow phases. The Galilee-consistent combination of words that are not closed to governing standards that need to be modeled are explained. Links between unclosed words in each mathematical template are available. Hybrid EE-LE statistics can benefit from this correspondence, which works consistently in the transmission of information. This analysis also serves as a guiding framework for direct numerical simulation of the two-phase flow that has been avoided in order to balance the unambiguous words in the dominant scales in these two mathematical presentations. Also, the benefits and limitations of these mathematical presentations are explained.

Keywords: PDF equation, Two Phase, Motion.

1. Introduction

In classical field theory the Lagrangian flow pathway is a method of analyzing the flow of fluid in which the analyst follows each liquid particle as it travels through space and time. By adjusting the position of each particle over time, we can analyze the structure of the particles. Although in the Eulerian path of the flow cycle it is a method of analyzing the movement of fluid concentrated in certain places in the space in which the fluid flows over time. The Lagrangian and Eulerian specifications of the flow are sometimes described as the Lagrangian and Eulerian reference framework. However, in most cases both the Lagrangian and Eulerian specifications of the flow can be applied to any viewing reference framework and to any communication system used within the selected reference framework. In the Eulerian specification field, it is represented as state function x and time t. For example, the flow rate is represented by the function v = v (x

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(t), t) = v (x, t). On the other hand, in the Lagrangian specification, individual liquid particles are tracked over time. Liquid particles are labeled with a temporary vertical vector x_0. Here x_0 can be considered as the center of particle weight at some point in the first t_0.

Two-phase flow statistical presentations are usually divided into Eulerian – Eulerian (EE) or Lagrangian – Eulerian (LE) methods depending on the reference frames under which they are built. EE statistical representation refers to a mathematical approach in which both continuous and scattered phases are defined in the standard Eulerian reference framework as random Eulerian fields. The mathematical representation of LE is a mathematical representation of the dispersed class in the Lagrangian framework in terms of density based on the location of the dispersed class (DPE) institutions [1].

The mathematical representations of EE and LE are basically the definition of the flow of two phases in two reference frames, it is natural to expect that these presentations are related. A major challenge in defining the flow of two phases is, therefore, establishing a direct relationship between these two modeling methods. Now, in taking the circumstances of the stated relationship under which it is held and the circumstances in which it does not need to be explicitly stated. By finding the right type of relationship between the presentations of two figures given certain practical conclusions. In 1998, Subramaniam et al. [6] suggested that the calculation of two-phase applications such as fuel sprays may be achieved by using the EE simulation method in the vicinity of the nozzle and the LE method in the dispersed spray area. This can be illustrated by Figure 1 showing the diagram of the EE-field system, LE-field and the distribution from the representation of EE to the representation of LE in the spray [1].

In the EE method, the two-phase flow field is represented as a random field while in the LE path the dispersed phase is represented as a marked point system embedded in the network company flow. The underlying events and their associated opportunities related to the flow of two phases in the EE and LE framework were discussed in this analysis.

2. Random Field Representation

Let us consider a realization of a two phase flow with two distinct thermodynamic phases i.e. a carrier phase and a dispersed phase. Furthermore the term 'two phase flow' will be taken as an isothermal two phase flow without any reactions. Now each realization can be taken as an element of some sample space D that is the space of all possible real events. In a single realization and at a single space-time location, the phases are considered by using an indicator function $I_\alpha(x, t)$ for the α^{th} phase and will be defined as:

$I_\alpha(\boldsymbol{x}, t)$ =1, if $\boldsymbol{x}$ is in phase α at time *t;*
=0, if $\boldsymbol{x}$ is not in phase α at time *t.*

In two phase flows, the phase indicator functions satisfy the relation

$$\sum_{\alpha=\{c,d\}} I_\alpha(x,t) = 1. \qquad (1)$$

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Where c represents the carrier phase and d represents the dispersed phase, for all the values (x, t). The instantaneous two phase velocity field $U(x, t)$, which is defined in both phases, is a vector field that is defined at each point x in the flow domain in physical space D.

The term $\rho(x, t)$ is the thermodynamic mass density function and is defined in both the phases. It has been considered that the density difference between the two phases is sufficiently large so that the density field can be used to distinguish between the two phases and the characteristic length scale of the interface over which this density change occurs is so small that in a continuum description the density changes discontinuously at the interface. But the phases are distinguished by the indicator function only then no information on shape or number of dispersed phase elements is available in this approach. Now the different events can be used to characterize the state of a two phase flow at a single space-time location (x, t), and each leads to different probabilities and p.d.f.'s. A complete Eulerian single-point p.d.f. description of the two phase flow will need the knowledge of the event i.e.

$$E_1 = [U \in (u, u + \delta u), \text{if } (x, t) = 1] \quad (2)$$

Which is the event corresponding to the joint occurrence of U in the range $(u, u + \delta u)$ at any point x and the fluid phase will present at the same point. Here u is the sample space variable corresponding to the random variable U. It will be noted that $I_c(x, t) = 1$ automatically precludes the occurrence of the dispersed phase at that same point i.e., $I_d(x, t) = 0$ at the same point x. Now for the joint event E_1, the two marginal events are

$$E_2 = [U(x, t) \in (u, u + \delta u)] \quad (3)$$

$$E_3(\alpha) = [I_\alpha(x, t) = 1] \quad (4)$$

Where E_2 is the event that $U(x, t)$ belongs to $(u, u + \delta u)$ without concern for whether the phase α is located at x, while $E_3(\alpha)$ is the event that phase α exists at x. The two conditional events are also useful and important is given by

$$E_4 = [U(x, t) \in (u, u + \delta u) | I_\alpha = 1] \quad (5)$$

$$E_5 = [I_\alpha(x, t) = 1 | U = u] \quad (6)$$

Where E_4 is the event that $U(x, t)$ belongs to $(u, u + \delta u)$ conditional on the presence of phase α at a point x, while E_5 is the event that the location x is occupied by the phase α with respect to condition on $U = u$ at the same location. Let the Eulerian p.d.f. of U be denoted as $f_U(u; x, t)$, where x and t are parameter space variables. The probabilities corresponding to each of the above events are given by

$$P[E_2] = P[U(x, t) \in (u, u + \delta u)] = f_U(u; x, t)\, \delta u \quad (7)$$

$$P[E_5] = P[I_\alpha(x, t) = 1 | U = u] = P_\alpha(x, t|u) \quad (8)$$

$$P[E_1] = P[I_\alpha(x, t) = 1 | U = u]\, P[U(x, t) \in (u, u + \delta u)] = P_\alpha(x, t|u)\, f_U(u; x, t)\, \delta u \quad (9)$$

$$P[E_3(\beta)] = P[I_\alpha = 1 | U = u]\, f_U(u)\, \delta u = P_\alpha f_U(u)\, \delta u = A_\alpha(x, t) \quad (10)$$

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$$P[E_4] = P[U(x, t) \in (u, u + \delta u)|I_\alpha = 1] = P_\alpha f_U(u; x, t)\, A_\alpha(x, t)\, \delta u, \quad (11)$$

Here $P_\alpha(x, t|u)$ is a phase probability function. Also, $A_\alpha(x, t)$ is the volume fraction at (x, t). The probability $P[E_3(\alpha)]$ defines a probability field $A_\alpha(x, t)$ is given as:

$$A_\alpha(x, t) \equiv P[I_\alpha(x, t) = 1] \quad (12)$$

Here $A_\alpha(x, t)$ is not a probability density function in x. It is a probability mass function in I_α that takes values $\{0, 1\}$. But f_U is a p.d.f. then it has to satisfy the condition of normalization i.e.,

$$\int f_U(u; x, t)\ \delta u = 1. \quad (13)$$

Also, let the probability $P[E_4]$ be denoted by $f_U|I_\alpha\ \delta u$, so that the Eulerian p.d.f. of velocity conditioned on the presence of phase α at x, $f_U|I_\alpha$ is given as:

$$f_U|I_\alpha = \frac{P_\alpha f_U(u)}{A_\alpha(x, t)} \quad (14)$$

The phase probability function P_α and the p.d.f. of instantaneous two phase velocity f_U can be written in terms of the volume fraction field A_α and the phasic velocity p.d.f. $f_U|I_\alpha$ as follows:

$$P_f(x, t|u) = \frac{A_c(x, t) f_{U|I_c}}{A_c(x, t) f_{U|I_c} + A_d(x, t) f_{U|I_d}} \quad (15)$$

$$f_U(u; x, t) = A_c(x, t) f_{U|I_c} + A_d(x, t) f_{U|I_d} \quad (16)$$

Hence these equations are sufficient for a complete single point description of a two phase flow motion.

3. Point Process Representation

In 1958, William et al., Gave the definition of LE a two-phase flow [7]. The spray equation, which is the equation equation of p.d.f., Can be accurately detected by starting with the Lagrangian equation equation of droplet position, velocity and radius [3], [4]. But d.d.f. began to define the fuel spray on internal combustion engines and hence the term 'droplet' distribution function was coined. It can be used to describe any two-phase flow where a dispersed phase can be modeled as a set of different components. Let's consider DPEs as drops, but conversation works equally well with other DPEs. And let us consider the flow of two phases in a D-flow zone in the space visible as a set of droplets. It is thought that one could associate the scale of the element with each radiant drop in the form of circular drops. If the droplet is not spherical, we will then use the same sphere radius with the same volume as the non-spherical droplet. We can also use the drop volume directly as a phase spacing. However, any choice does not automatically change the detection of the spray number and does not provide further details about the nature of the words that are not closed in the spray count and the time figures obtained from it. Thus, we retain radius as a measure of the length of the space element of the size section [5].

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At time t the total number of droplets $n(t)$ is a non-negative integer valued random variable that is finite with probability 1. The i^{th} DPE is characterized by its position vector $x_i(t)$ which is defined as the centre of mass of the droplet, its velocity vector $v_i(t)$ and its radius $r_i(t)$. The position, velocity and radius of a droplet are called the droplet properties and the droplet property vector associated with each droplet is a seven-dimensional random vector in this representation. Any other additional droplet properties may be included as required but they do not fundamentally alter the formulation other than increasing the dimension of the space of droplet properties. The properties associated with the i^{th} droplet given by the following equations:

$$\frac{dx_i}{dt} = v_i, \frac{dv_i}{dt} = a_i, \frac{dr_i}{dt} = \delta r_i \qquad (17)$$

Where a_i is the acceleration experienced by the droplet, and δr_i is the rate of radius change due to vaporization or due to other reasons. This initial physical description for the LE approach assumes that the velocity field inside the droplet is uniform, and hence the motion of the i^{th} droplet can be described by the motion of its centre of mass x_i. The ensemble of droplets is characterized in the seven-dimensional position velocity radius space (x, v, r) by its fine grained density function γ, which is defined as

$$\gamma(x,v,r,t) = \sum_{i=1}^{n} \gamma_i = \sum_{i=1}^{n} \delta(x - x_i)\ \delta(v - v_i)\ \delta(r - r_i) \qquad (18)$$

Here (x_i, v_i, r_i) are the Lagrangian coordinates of the i^{th} droplet whereas (x, v, r) are the sample-space coordinates. The function γ represents the density of droplets in a seven-dimensional (x, v, r) space. The summation of the product of delta functions in the above equation represents a single realization of the two phase flow. So the above summation represents a realization of the two phase flow in which the i^{th} DPE whose centre of mass x_i is at location x in position phase space, whose centre of mass velocity v_i is at location v in velocity phase space and whose radius r_i is at location r in radius phase space. If the number of droplets in any region B^+ in (x, v, r^+) space (since droplets with only non-zero radius belong to the spray system, if we denote r^+ to be the positive r-axis ($r > 0$), then it is suitable to integrate over regions only in (x, v, r^+) space) is denoted by $n(B^+; t)$, it is obtained by integrating γ over the region B^+ such that

$$n(B^+; t) = [\int \gamma(x,v,r,t)\ dx\, dv\, dr]_{B^+} \qquad (19)$$

But γ is composed of delta functions it is not a smooth function in (x, v, r) space. The statistical description of a spray in terms of γ contains far more information than that is necessary for the calculations. So in order to find the information concerning the average properties of the spray, it is advantageous to consider the whole total average of γ. This average of γ is denoted by (x, v, r, t) and it defines the d.d.f. as given

$$\gamma_a(x, v, r, t) \equiv \langle\gamma(x,v,r,t)\rangle = \langle\textstyle\sum_{i=1}^{n} \delta(x - x_i)\ \delta(v - v_i)\ \delta(r - r_i)\rangle \qquad (20)$$

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The expectation $\langle . \rangle$ in the above represents a whole total average of possibly infinite realizations of the two phase flow. The details on the use of the delta function to represent a realization of a single phase flow and its whole total average can be found. It is important to note that the expectation operator cannot be brought inside the summation for a general spray; if done, and then the conclusions of such an operation needs to understand [3].

The expected number of droplets $n(B^+; t)$ in a region B^+ of (x, v, r^+) space is given by

$$n(B^+; t) = \int_{B^+} \gamma_a (x, v, r, t) dx\, dv\, dr \tag{21}$$

The expected number of spray droplets $\langle n(t) \rangle$ at time t over the entire space (x, v, r^+) is given by

$$\langle n(t) \rangle = \int_{(x,v,r^+)} \gamma_a (x, v, r, t) dx\, dv\, dr \tag{22}$$

If the droplet distribution function is integrated over only (v, r^+) space, the density of the expected number of spray droplets $n_e(x; t)$ can be finding as follows:

$$n_e(x; t) = \int_{(v,r^+)} \gamma_a (x, v, r, t)\, dv\, dr \tag{23}$$

The joint p.d.f. i.e., j.p.d.f. for a p.d.f. conditional on x of velocity and radius can be taken as:

$$\gamma_{vr}^{x}(v, r; x; t) = \frac{\gamma_a(x, v, r, t)}{n_e(x; t)} \tag{24}$$

Let us take,
$P[n(t)=k] = p_k$, γ^k denotes the density of expected number of droplets in the phase space with respect to the condition that the event $n(t)=k$ and $\gamma_{1s}^{k}(x, v, r; t)$ is the single-particle density of identically distributed considered droplets then we have

$$\gamma_a(x, v, r, t) = \sum_{k\geq 1} p_k \gamma^k (x, v, r; t) = \sum_{k\geq 1} k\, p_k \gamma_{1s}^{k} (x, v, r; t) \tag{25}$$

Hence we have the relation as given

$$\gamma_{vr}^{x}(v, r; x; t) = \frac{\sum_{k\geq 1} k\, p_k \gamma_{1s}^{k} (x, v, r; t)}{\sum_{k\geq 1} k\, p_k \gamma_{1s}^{k} (x; t)} \tag{26}$$

4. Conclusions

Two distinctly mathematical presentations, namely the Eulerian-Eulerian and Lagrangian – Eulerian mathematical presentations, exist in the flow of two phases. It is clearly shown that the possible representations of EE and LE of the two-phase flow have a complex, distinct relationship than the simple relationship between Eulerian and Lagrangian definitions in a single

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phase flow. Significant events and related opportunities related to the two-phase flow in the statistical representation of EE have been established. The control figures for the average weight, pressure ratio and the second time based on the given EE mass density calculations are shown to correspond to the general use of the sum of the total median flow values of the two phases. Central to the mathematical representation of LE is the function of the droplet distribution (d.d.f.) whose measurement of occurrence is accurately calculated with the aid of point processes. The transport figure forms the basis of the average weight gain, the intensity of the measurement and the second-minute calculations of the dispersed phase in the representation of LE. Conformity conditions are established between EE values and LE statistical representations. By comparing the unambiguous words in the median numerical controls, the intensity of the measurement and the second minute in each statistical representation, a connection between the unambiguous words is established. Comparisons between two statistical presentations show that the content of the two approaches is very different.

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