

# Two Phase Flows Motions Regarding Wave Equations in Fluid Flow Theory

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## Abstract

The purpose of the present paper is to discuss weak linear waves using a mixture of two phases of gas and dust particles, in which particle-volume-fraction appears as an additional variable. Solutions based on asymptotic process are obtained under the assumption that the length of the signal element is much shorter than the length of the local feature.

**Keywords:** Distribution, Two categories, gas particles.

## 1. Introduction

In the latest technological advances in various fields of engineering and science, the pressurized flow of dusty gas is being met. When a gas carries a large amount of solid particles, a two-phase relaxation event significantly affects the flow field. It has been shown by Lick (1967) and Parker (1969) that a time-varying disruption of the reduction period can cause a slightly dispersed and fully dispersed shock in the resting gases. However in some physical conditions a signal with a shorter duration than duration may be further disturbed leading to the formation of shock. In some cases this catastrophic effect is delayed due to dispersion but in non-dispersed systems it occurs due to non-integration as long as the transmission distances are large enough (1975, 1971, 1972). A common discussion of low amplitude waves with the consideration of indirect effects is illustrated in the works of Light-hill (1949), Witham (1952) and Lin (1955).

It is known that gas flow with an optimal number of small solid particles may show results of greater relaxation, due to the inability of the particles to follow rapid changes in the speed and temperature of the gas. Such effects are more common in the presence of tidal waves in dusty gases. The problem of acoustical damming in dusty gases is addressed by Epstein by



direct analysis (1955, 1941). Bhutani and Chandran (1977) negotiated weak dust waves using a system of integration; have analyzed aircraft decay, cylindrical and spherical weak-waves in the gas particle system.

The purpose of the paper is to discuss weak linear waves using a mixture of two layers of gas and dust particles, where the particle-volume particle appears as an additional variable. Solutions based on simple asymptotic wave processes, are found under the assumption that the length of the signal element is much shorter than the length of the local element.

## 2. Basic statistics and boundary conditions

The mathematical analysis of the two-phase flow is much more complex than the pure gas flow, and one of the most common assumptions that make it easier for particle volume to be ignored. At high gas levels (high pressure) or at large particles, the fractional particle volume may be large enough to be built under the following predictions-

1. Gas complies with the complete law of gas and certain temperatures do not change.
2. Particles are round, of the same size and distributed evenly at the beginning. The exact temperature does not change and the temperature is the same within each particle.
3. Particles do not mix and their motion is absent.
4. The viscosity and temperature of the gas are ignored without contact with solid particles.
5. Particles do not apply pressure.
6. No external force (such as gravity) or temperature fluctuations affecting the mixture and no significant transfer of gas between particles and particles.

Statistics controlling the movement of gas particles, under the above speculation are provided by (1969).

$$u_{,x} + uu_{,x} + \frac{RT}{\rho(1-\epsilon)} \rho_{,x} + \frac{R}{(1-\epsilon)} T_{,x} + \frac{\epsilon \rho_p}{\rho(1-\epsilon)^2} \frac{(u-v)}{\tau_v} = 0, \quad (2.1)$$



$$\rho_{,t} + \rho u_{,x} + u \rho_{,x} + \frac{\epsilon \rho}{(1-\epsilon)} v_{,x} + \frac{\rho}{(1-\epsilon)} (v-u) \epsilon_{,x} = 0, \quad (2.2)$$

$$T_{,t} + (\gamma-1) T u_{,x} + \frac{T \epsilon (\gamma-1)(v-u)}{\rho (1-\epsilon)} \rho_{,x} + \frac{[u(1-\gamma\epsilon) + \epsilon v(\gamma-1)]}{(1-\epsilon)} T_{,x} \\ + \frac{\epsilon T (\gamma-1)}{(1-\epsilon)} v_{,x} + \frac{T (\gamma-1)(v-u)}{(1-\epsilon)} \epsilon_{,x} + \frac{\epsilon \rho_p}{(1-\epsilon) \rho C_v} \left[ \frac{C_m (T - T_p)}{\tau_T} - \frac{(u-v)^2}{(1-\epsilon) \tau_v} \right] = 0, \quad (2.3)$$

$$v_{,t} + v v_{,x} + \frac{(v-u)}{(1-\epsilon) \tau_v} = 0, \quad (2.4)$$

$$\epsilon_{,t} + \epsilon v_{,x} + v \epsilon_{,x} = 0, \quad (2.5)$$

$$T_{p,t} + v T_{p,x} \frac{(T_p - T)}{\tau_T} = 0, \quad (2.6)$$

where  $p, \rho, T, u, C_p, C_v, \gamma$  be the pressure, density, temperature, velocity, specific heats and specific heat ratio of the gas and  $\epsilon, T_p, v, C_m$  be the volume-fraction, temperature, velocity and specific heat of the dust particles respectively.  $\tau_v$  is relaxation time for particle velocity and  $\tau_T$  is relaxation time for heat transfer. A Comma followed by an index denotes the partial differentiation with respect to index.

### 3. Characteristic Equations

In studying the wave phenomenon governed by hyperbolic equations, it is usually more natural and convenient to use the characteristics of governing system as the reference coordinate system. Let us introduce the characteristic variables ' $\alpha$ ' and ' $\psi$ ' such that;

$$\alpha_{,t} + u \alpha_{,x} = 0, \quad (3.1)$$

$$\psi_{,t} + (u+a) \psi_{,x} = 0, \quad (3.2)$$



The leading characteristic front can be represented by  $\alpha = 0$  and if a gas particle crosses this front at time 't' its path will be represented by  $\psi = t$ . Keeping in view of the properties of  $\alpha$  and  $\psi$ , it is obvious that the function  $x(\alpha, \psi)$  and  $t(\alpha, \psi)$  satisfy the following partial differential equations;

$$x_{,\alpha} = ut_{,\alpha}, \quad x_{,\psi} = (\underline{u} + \underline{a})t_{,\psi}$$

The transformation from space time (x,t) to the plane of characteristic parameters  $(\alpha, \psi)$  will be one to one if and only if Jacobian

$$J = (x_{,\alpha} t_{,\psi} - x_{,\psi} t_{,\alpha}) = (u - \underline{u} - \underline{a})t_{,\alpha} t_{,\psi}$$

or

$$J = \left( \frac{\epsilon(\gamma - 1)(u - v)}{2(1 - \epsilon)} - \underline{a} \right) t_{,\alpha} t_{,\psi},$$

does not vanish or does not become infinity anywhere.

Since  $t_{,\psi} \neq 0$ , from physical considerations a breakdown of solution in terms of characteristic parameters will arise if and only if  $t_{,\alpha} = 0$ ,

In terms of characteristic coordinates equations given by (2.7) reduces to the following form.

$$\begin{aligned} & \left[ \frac{\epsilon(\gamma - 1)(u - v)}{2(1 - \epsilon)} - \underline{a} \right] u_{,\alpha} t_{,\psi} + \frac{1}{\gamma(1 - \epsilon)} (T_{,\alpha} t_{,\psi} - T_{,\psi} t_{,\alpha}) \\ & + \frac{T}{\rho\gamma(1 - \epsilon)} (\rho_{,\alpha} t_{,\psi} - \rho_{,\psi} t_{,\alpha}) \\ & + \left[ \frac{\epsilon(\gamma - 1)(u - v)}{2(1 - \epsilon)} - \underline{a} \right] \frac{\epsilon p_p}{(1 - \epsilon)^2 p} (u - v) t_{,\alpha} t_{,\psi} = 0, \end{aligned} \tag{3.5}$$

$$\left[ \frac{\epsilon(\gamma-1)(u-v)}{2(1-\epsilon)} - \underline{a} \right] p_{,\alpha} t_{,\psi} + p \left[ u_{,\alpha} t_{,\psi} - u_{,\psi} t_{,\alpha} \right] + \frac{\rho(v-u)}{(1-\epsilon)} \left[ \epsilon_{,\alpha} t_{,\psi} - \epsilon_{,\psi} t_{,\alpha} \right] + \frac{\epsilon p}{(1-\epsilon)} \left[ v_{,\alpha} t_{,\psi} - v_{,\psi} t_{,\alpha} \right] = 0, \quad (3.6)$$

$$\begin{aligned} & \frac{\epsilon(\gamma-1)(u-v)}{(1-\epsilon)} T_{,\psi} t_{,\alpha} + \left[ \frac{\epsilon(\gamma-1)(u-v)}{2(1-\epsilon)} - \underline{a} \right] T_{,\alpha} t_{,\psi} \\ & + \frac{T(\gamma-1)(u-v)}{(1-\epsilon)} \left[ \epsilon_{,\alpha} t_{,\psi} - \epsilon_{,\psi} t_{,\alpha} \right] \\ & + T(\gamma-1) \left[ u_{,\alpha} t_{,\psi} - u_{,\psi} t_{,\alpha} \right] + \frac{\epsilon T(v-u)(\gamma-1)}{(1-\epsilon)\rho} \left[ \rho_{,\rho} t_{,\psi} - \rho_{,\psi} t_{,\alpha} \right] \\ & + \frac{\epsilon T(\gamma-1)}{(1-\epsilon)} \left[ v_{,\alpha} t_{,\psi} - v_{,\psi} t_{,\alpha} \right] + \frac{\epsilon \rho_{\rho} \gamma}{(1-\epsilon)\rho} \left[ \frac{\epsilon(\gamma-1)(u-v)}{(1-\epsilon)} - \underline{a} \right] \\ & \left[ \eta(T - T_p) - \frac{(\gamma-1)(u-v)^2}{(1-\epsilon)} \right] t_{,\alpha} t_{,\psi} = 0, \end{aligned} \quad (3.7)$$

$$\begin{aligned} & (u-v)v_{,\psi} t_{,\alpha} + \left[ \frac{(u-v)\{\epsilon(\gamma+1)-2\}}{2(1-\epsilon)} - \underline{a} \right] v_{,\alpha} t_{,\psi} \\ & + \left[ \frac{\epsilon(\gamma-1)(u-v)}{2(1-\epsilon)} - \underline{a} \right] \frac{(u-v)}{(1-\epsilon)} t_{,\alpha} t_{,\psi} = 0, \end{aligned} \quad (3.8)$$

$$\begin{aligned} & (u-v)v_{,\psi} t_{,\alpha} + \left[ \frac{(u-v)\{\epsilon(\gamma+1)-2\}}{2(1-\epsilon)} - \underline{a} \right] \epsilon_{,\alpha} t_{,\psi} \\ & + \epsilon \left( v_{,\alpha} t_{,\psi} - v_{,\psi} t_{,\alpha} \right) = 0, \end{aligned} \quad (3.9)$$



$$(u-v)T_{p,\psi}t_{,\alpha} + \left[ \frac{(u-v)\{\epsilon(\gamma+1)-2\}}{2(1-\epsilon)} - a \right] T_{p,\alpha}t_{,\psi} + \Gamma \left[ \frac{\epsilon(u-v)(\gamma-1)}{2(1-\epsilon)} - a \right] (T_p - T)t_{,\alpha}t_{,\psi} = 0. \quad (3.10)$$

If we consider the case in which the wave front is an outgoing characteristic propagating into a uniform region, boundary conditions are given by

$$u = v = 0, \rho = 1, T = 1 = T_p \text{ and } \epsilon = \epsilon_0, \text{ at } \alpha = 0$$

$$\text{and } u = F'(\alpha), x = F(\alpha), t = \alpha, \text{ at } \psi = 0.$$

#### 4. Conclusions

The purpose of the paper is to discuss weak linear waves using a mixture of two layers of gas and dust particles, where the particle-volume particle appears as an additional variable. Solutions based on simple asymptotic wave processes, are found under the assumption that the length of the signal element is much shorter than the length of the local element.

#### References

- [1]. Epstein, P. S., and Carhrat, R. R., 1955; *J. Acoust. Soc. Am.* 25, 555.
- [2]. Epstein, P. S., 1941; Contributions to Applied Mechanics, Theodore Von Marman Anniversary Volume, *Calif. Inst. Technol.*, 162.
- [3]. Lick, W.J., 1967; In Advance of Applied Mechanics 10, Academic, New York.
- [4]. Lighthill, M. J., 1949; *Phil. Mag.* 50, 1179.
- [5]. Lin, C. C., 1955; *J. Maths. Phys.* 33, 117.
- [6]. Parker, D. F., 1973; *Physics Fluids*, 15, 256.
- [7]. Parker, D. F., 1971; *J. Inst. Math. Applice* 7, 92.
- [8]. Parker, D. F., 1969; *J. Fluid Mech.* 39, 793.



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- [9]. Seymour. B. R. and Mortell, M. P., 1965; *Non Linear Geometrical Acoustics Mechanic Today 2*, Pergamon Press.
- [10]. Witham, G. B., 1952; *Comm. On Pure and Appl. Maths.*,301.

