Journal of Advance Research in Science and Social Science (JARSSC) **ISSN:** 2582-2004
Official Publication of Indian Mental Health & Research Centre Volume 05, Issue 01

DOI: 10.46523/jarssc.05.01.15
Multidisciplinary, Open Access

GRADED PRIMARY AND GRADED SECONDARY MODULES

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Abstract

Let R be a G- graded commutative ring and G be a group with identity e. in this paper we show that every graded primary submodule of a graded representable module over a G-graded ring is graded representable and here we study graded representable module and the graded primary submodules of a graded module over a G-graded commutative ring.

Keywords: Commutative ring, graded primary, graded secondary modules.

Introduction

Secondary modules have been studied extensively by many authors (Atani 2006, Macdonald 1973, Nastasescu 1982). Here we examine when graded submodules of a graded representable module are graded representable a number of results concerning of this class of submodules. Various properties of such modules are considered.

Before we state some results let us introduce some notation and terminology. Let G be an arbitrary group with identity e. A commutative ring R with non-zero identity is a graded if it has a direct sum decomposition (as an additive group) $R = \oplus_{g \in G} R_g$ such that $1 \in R_e$; and for all g, h $\in$ G, $R_g R_h \subseteq R_{gh}$. If R is G-graded, then an R-module M is said to be G-graded if it has a direct sum decomposition $M = \oplus_{g \in G} M_g$ such that for all g, h $\in$ G, $R_g M_h \subseteq M_{gh}$. An element of some R_g or M_g is said to be homogeneous element. A submodule of N $\subseteq$ M, where M is G-

Received: 26.01.2022 **Accepted:** 05.03.2022 **Published**: 07.03.2022

DOI: 10.46523/jarssc.05.01.15
Multidisciplinary, Open Access

graded, is called G-graded if $N = \oplus_{g\in G}(N\cap M_g)$ or if, equivalently, N is generated by homogeneous elements.

Moreover, M/N becomes a G-graded module with g-component $(M/N)_g = (M_g+N)/N$ for $g \in G$. Clearly, 0 is a graded submodule of M. Also, we write $h(R) = \cup_{g\in G}R_g$ and $h(M)= \cup_{g\in G}M_g$. A graded ideal I of R is said to be a graded prime ideal if $I \neq R$; and whenever $ab\in I$, we have $a\in I$ or $b\in I$, where $a, b\in h(R)$. The graded radical of I, denoted by Gr(I), Is the set of all $x\in R$ such that for each $g\in G$ there exists $n_g>0$ with $x_g^{n_g} \in I$. A graded ideal I of R is said to be a graded primary ideal if $I \neq R$; *and whenever* $a,b \in h(R)$ with $ab\in I$, then $a\in I$ or $b\in Gr(I) = P$ is a graded prime ideal of R, and we say that I is a graded P-primary ideal of R (Refai, 2004).

Let S be a commutative ring and let M an R-module. Given an element *a* of S, we say that a divides M if aM = M, and we say that a is nilpotent on M if $a^nM = 0$ for some n, We say that M is secondary if it is non-zero and every $a\in S$ either divides M or is nilpotent on M; in this case the ideal rad(M) = P is prime and we also say that M is P-secondary (Macdonald, 1973).

2. Graded primary submodules

First, we give some basic basic facts concerning graded primary submodules of a graded module. Next, we study graded submodules of a graded representable module.

Definition 2.1 Let R be a G-graded ring, M a graded R-module and N a graded R-submodule of M.

(i) We say that M is a graded free R-module if it has an R-basis consisting of homogeneous elements.

(ii) N is a graded prime submodule of M if $N \neq M$; and whenever $a \in h(R)$ and $m \in h(M)$ with $am \in$ **N,** then either $m \in N$ or $a \in (N:_R M)$.

(iii) N is a graded primary submodule of M if $N \neq M$; and whenever $a \in h(R)$ and $m\in h(M)$ with $am \in N$, then either $m \in N$ or $a^k \in (N:_R M)$ for some k.

Received: 26.01.2022 **Accepted:** 05.03.2022 **Published**: 07.03.2022

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(iv) N is a graded maximal submodule of M if N ≠ M and there is no graded submodule K of M such that $N \subsetneq K \subsetneq M$.

(v) We say that M is graded simple module if it has only two graded submodules 0 and M.

The following lemma is known, but we write it here for the sake of reference.

Lemma 2.2 *Let R be a G-graded ring, M a graded R-module and N a graded R-submodule of M. Then the following hold:*

(i) N is a graded maximal submodule of M if and only if M/N is a graded simple R-Module.

(ii) If r∈h(R), x∈h(M) and I is a graded ideal of R, then $(N:_R M)$ is a graded ideal of R, Rx, IN and r N are graded submodules of M.

The graded radical (resp. Radical) of a graded submodule (resp. submodule) N of a graded module (resp. module) M, denoted by Gr (N), (resp. rad(N)) is defined to be intersection of all graded prime (resp. prime) submodules of M containing N. Clearly, if N and K are graded submodules of M with K⊆N, then Gr(K) ⊆ Gr(N). Let M be zero-divisor on M if there exists 0 ≠ m ∈M such that rm = 0.

Lemma 2.3 *Let M be a graded simple module over a G-graded ring R. Then every graded zero-divisor on M is an annihilator of M*

Proof. Let r be an arbitrary graded zero-divisor on M. Then there exists 0 ≠ a∈ h(M) such that ra=0. Since M is a simple graded R-module, we get Ra = M. Hence rM = r(Ra) = (Rr)a = R(ra) = 0. Thus, r is an annihilator of M.

Proposition 2.4 Let M be a graded module over a G-graded ring R. Then every graded Maximal submodule of M is a graded prime.

Proof. Let N be an arbitrary graded maximal submodule of M. Let r m∈N where r∈h(R)and m∈h(M) – N. Since 0 ≠ (m+N) ∈h(M/N) and r(m+N) = 0, we get r is a graded zero-divisor on graded module M/N; hence by lemma 2.2 and lemma 2.3, $r\in(N:_R M)$, as required.

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Proposition 2.5 Let R be a G-graded ring, M a graded R-module and N a graded R-submodule of M. Then the following hold:

(i) If N is a graded primary submodule of M, then $(N:_R M)$ is a graded primary ideal of R.

(ii) If N is a graded prime submodule of M, then $(N:_R M)$ is a graded prime ideal of R.

Proof. (i) Clearly, $(N:_R M) \neq R$. Let $ab \in (N:_R M)$ with $b \notin (N:_R M)$ where $a, b \in h(R)$, So there exists $m \in h(M) - N$ such that $bm \notin N$. As $abm \in N$, N graded primary gives $a^k M \subseteq N$ for some k, as needed.

(ii) The proof is similar to that of (i).

Proposition 2.6 Let R be a G-graded ring, M a graded free R-module and I an ideal of R. Then the following hold.

(i) If I is a graded primary ideal of R, then IM is a graded primary submodule of M.

(ii) If I is a graded Prime ideal of R, then I M is a graded prime submodule of M.

Proof. (i) As M is a cancellation module and $I \neq R$, we get $I M \neq M$, Assume that M is the graded free R-module with a homogeneous basis $\{x_g : g = G\}$ and let $rm \in I M$ with $m \notin I M$ where $r \in h(R)$ and $m \in h(M)$. We can write $m = \sum_{i=1}^{n} r_i x_{g_i}$ *with* $r_i \in R$, Since $M \notin I M$, there exists an integer J such that $r_j \notin I$. There are elements $b_1, \ldots, b_n \in I$ such that $\sum_{i=1}^{n} (rr_i) x_{g_i} = \sum_{i=1}^{n} b_i x_{g_i}$, so $rr_i = b$, for every $i = 1, \ldots, n$; hence $rr_j \in I$.

Since $r_j = \sum_{i=1}^{n} r_{gi} \notin I$ with $r_{g_i} \neq 0$, we obtain that $r_{g_t} \notin I$ for some t. It follows that $rr_{g_i} \in I$ since I is graded ideal, so $r^m \in I$ for some m; hence $r^m M \subseteq IM$, as required.

(ii) The proof is similar to that of (i)

One approach to the graded case is simply to redefine all of the treminology to involve only homogeneous elements and graded submodules. In this veiw, a non-zero graded module M is graded secondary if every homogeneous element of R either divides M or is nilpotent on M, in

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which case Gr(annM) = P is a graded prime ideal of R, and M is said to be graded P-secondary. A graded module M is said to be graded secondary representable if it can be writen as a sum M = M_1+....+M_K with each M_i *graded* secondary, and if such a representation exists (and is irredundant) then the graded attached primes of M are Att (M) = {Gr(annM),....,Gr(annM)}. Note that a graded secondary module, in general, is not secondary. For example, as discussed in (Sharp, 1986), if R = $k[x]$ is a polynomial ring in one variable with the natural Z-graded ring and M = $k[x,1/x]$, then M is graded secondary but is not secondary. so the graded secondary and secondary modules are different. concepts.

A graded submodule N of M is said to be graded pure submodule if aN = N$\cap$ aM for every a $\in$h(R). We have the following proposition.

Proposition 2.7 Let R be a G-graded ring, M a graded R-module and N a non-zero graded pure R-submodule of M. Then M is a Graded P-secondary if and only if both N and M/N are graded P- secondary.

Proof. Assume that M is P-secondary and let a$\in$h(R). If a $\in$P, then $a^sN\subseteq a^sM = 0$ and a^s(M/N) = 0 for some s, so a is nilpotent on N and M/N. If a$\notin$P, then aN = N$\cap$aM = N and a(M/N) = M/N, so a divides N and M/N; Hence N and M/N are P-secondary. Conversely, assume that N and M/N are P-secoundary and let b$\in$ h(R). If b$\in$P, then b^t M$\subseteq$N and 0 = b^tN = N$\cap b^t$M = b^tM for some t, so b is nilpotent on M. if b$\notin$P, then N = bN = N$\cap$bM and b(M/N), so bM = M, as required.

Theorem 2.8 Let R be a G-graded ring, M a graded secondary R-module and N a non-zero graded P-prime R-submodule of M. Then N is graded P-secondary.

Proof. Assume that M is a graded Q-secondary R-Module and let r$\in$h(R). If r$\in$Q, then r^s N $\subseteq$ r^s M = 0 for some s, so r is nilpotent on N. Suppose that r$\notin$Q; we show that r divides N. so assume that a$\in$N. Then there exists b=$\sum_{i=1}^{t} b_{g_i}$ $\in$ M (with $b_{g_i} \neq 0$) such that a = rb. As N is graded, r$b_{g_i}$$\in$N for every i = 1,....,t, so for each i, N graded prime gives b_{g_i} $\in$ N; hence b$\in$N. It follows that r divides N, so N is a graded Q-secondary R-module.

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Now we need to show that P = Q. Since the inclusion P⊆Q is trivial, we will prove the reverse inclusion. Suppose that c = $\sum_{i=1}^{n}$ c_{h_i}∈Q with $c_{h_i} \neq 0$. Then there are integers m_i such that $c_{h_i}^{m_i}$ M =0 for i=1,.....,n since Q is graded and M is graded Q-secondary. As M ≠ N, there is an element $x=x_{g_1}+...+x_{g_n}$ ∈ M (with $x_{g_i} \neq 0$) such that $x_{g_w} \notin$ N for some w. Therefore, for each i = 1,...,n, $c_{h_i}^{m_i}$ x_{g_w}= 0 ∈ N, so N graded Prime gives c_{hi} ∈ P; hence c ∈ P, as required.

Lemma 2.9 *Let R be a G-graded ring, M a graded R-module and N a graded P-secondary R-submodule of M. Then the following hold.*

(i) If K is a graded primary submodule of M, then N ∩ K is graded P-secondary.

(ii) If K is a graded prime submodule of M, then N∩ K is graded P-secondary.

Proof. (i) Assume that a ∈h(R) and let a ∈ P. Then a^m (N ∩ K) ⊆ a^m N = 0 for some m, so a is nilpotent on N∩K. Suppose that a∉ P; we show that a divides N∩K. It suffices to show that N∩K⊆ a (N∩K). If b ∈ N∩ K, then b=am for some m=$\sum_{i=1}^{n}$ m_{g_i} ∈N with $m_{g_i} \neq 0$. Then for each i=1,....,s, am_{g_i} ∈K since K is a graded submodule of M. It follows that m_{g_i} ∈K for every i (otherwise, if $m_{g_i} \notin$ K for some j and a^s ∈ (K :$_R$ M) for some s,

then m_{g_j} ∈ N = a^s N ⊆ a^s M ⊆ K which is a contradiction), so m∈K; hence b∈a (N∩K) and the proof is complete.

Theorem 2.10 (i) Every graded primary submodule of a graded representable module over a G-graded ring is graded representable.

(ii) Every graded prime submodule of a graded representable module oer a G-graded ring is graded representable.

Proof. (i) Assaume that M = $\sum_{i=1}^{k}$ S_i is a miimal graded secondary representation of M with Att (M) = {P_1,.....,P_k} and let N be a graded P-primary submodule of M. There exists a submodule S_i, such that $S_1 \nsubseteq$ N since N ≠ M. First we show that P = P_1 Let a = a_{g_i} +...+a_{g_t} ∈ P_1 with $a_{g_i} \neq 0$. There are integers n_1,.......,n_t and a homogeneous element yh ∈ S_1 – N such that

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$a_{g_t}^{n_i}$ yh = 0 for every i, so N graded Primary given $a_{g_i} \in$ P for every i; Hence a ∈ P. Therefore, $P_1 \subseteq P$. For the other containment, suppose that there exises a homogeneous element $C_h \in P$ with $c_h \notin P_1$. Then $S_1 = c_h^s S_1 \subseteq c_h^s M \subseteq N$ for some s which is a contradiction. Thus, $P = P_1$ Likewise, if $s_j \subseteq N$ for $j \neq 1$, then $P = P_1 = P_j$ which is a contradiction. We will show that $S_i \subseteq N$ for i=2,....k. As $P \neq P_i$ we divide the proof into two cases:

Case 1 $P \nsubseteq P_i$

There exists a homogeneous element ph ∈ P with ph ∉ P_i. Then $S_i = p_h^t S_i \subseteq p_h^t M \subseteq N$ for some t.

Case 2 $P_i \nsubseteq P$

There is a homogeneous element $a_g \in P_i$ with $a_g \notin P$. Let $b = \sum_{i=1}^{m} b_{h_i} \in S_i$ with $b_{g_i} \neq 0$. Then there is an integer n such that $a_g^n \; b_{h_i} = 0 \in N$, so N graded primary gives $b_{h_i} \in N$ for i=1,....,m; hence b ∈ N. Thus $S_i \subseteq N$. It follows that $N = N \cap M = N \cap S_1 + \sum_{i=2}^{k} S_i$. Now the assertion follows form Lemma 2.9.

Corollary 2.11 Let R be a G-graded ring, M a graded representable R-module and N a graded primary (resp. Graded prime) R-submodule of M. Then Att (N) ⊆ Att (M).

Proof. This follows from Theorem 2.10

Let R be a G-graded ring. The graded dimension of R is defined as the supermum of all numbers n for which there exists a chain of graded prime ideals $P_0 \subseteq P_1 \subseteq P_1 \subseteq ... \subseteq P_n$ in R and it is dnoted by Gdim R. We say that R is a G-graded integral domain whenever a, b∈ h(R) with ab = 0 implies that either a = 0 or b = 0.

Lemma 2.*12* *Let P be a graded prime ideal of a G-graded ring R, M a graded R-module and* $\{N_i\}_{i \in I}$ *a family of graded prime R-submodules of M such that* $(N_i :_R M) = P$ *for every* $i \in I$. *Then* $\cap_{i \in I} N_i$ *is a graded prime submodule of M.*

Proof. The proof is straight forward.

Received: 26.01.2022 **Accepted:** 05.03.2022 **Published**: 07.03.2022

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Theorem 2.13 Let R be a G-graded integral domain with Gdim R = 1, M a graded representable R-module and N a graded primary R- submodule of M. Then Gr(N) is graded representable.

Proof. Consider the greded ideal ($K :_R M$) for any graded prime submodule K containing N. These ideals are graded prime by Proposition 2.5 and $N \subseteq K$ implies $(N :_R M) \subseteq (K :_R M)$; hence by [5, Proposition 1.2], $Gr(N:_R M) \subseteq Gr(N:_R M)$ for all such K. For any one of these prime submodules K, we generate the chain of graded prime ideals $0 \subset Gr(N:_R M) \subseteq (K:_R M)$ since by [5, Lemma 1.8], $Gr(N :_R M)$ is a graded Prime ideal of R. As Gdim R=1, we must have $Gr(N:_R M) = (K :_R M)$ for every graded prime submodule K containing N. By Lemma 2.12 $Gr(N) = \cap_{N\subseteq k} K$ is a graded prime submodule of M. Now the assertion follows form Theorem 2.10

Lemma 2.14 *Let R be a G-graded ring , M a graded R-module and N a graded rep-resentable R- submodule of M. Then if K is a graded Primary (Resp. graded prime) submodule of M, then N∩ K is graded representable.*

Proof. By Theorem 2.10, it suffices to show that N∩K is a graded primary submodule of N. Let $an \in N \cap K$ with $n \notin N \cap K$ where $a \in h(R)$ and $n \in h(N)$, so K graded primary gives $a^s M \subseteq K$ for some s; hence $a^s (N\cap K) \subseteq N$, as required.

Theorem 2.15 Let R be a G-graded ring, M a graded R- Module and N a graded R- submodule of M such that N possess a graded primary decomposition. If K is a graded representable submodule of M, then N∩ K can be expressed as an intersection of finitely many graded representable submodules.

Proof. Let $N=\cap_{i=1}^{n} N_i$ where N_i is graded Primary, be a normal decomposition. Then $N\cap K= (N_1 \cap K) \cap ... \cap (K\cap N_n)$. Now the assertion follows form Lemma 2.14.

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Received: 26.01.2022 **Accepted:** 05.03.2022 **Published**: 07.03.2022

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DOI: 10.46523/jarssc.05.01.15
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Crossref DOI Prefix 10.46523
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Received: 26.01.2022 **Accepted:** 05.03.2022 **Published**: 07.03.2022